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CP VIOLATION AND THE WIDTH $Z \to b\bar{b}$

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Abstract:

We discuss the effect of CP–violating $Zb\bar{b}$, $Zb\bar{b}G$ and $Zb\bar{b}\gamma$ couplings on the width $\Gamma(Z \to b\bar{b}X)$. The presence of such couplings leads in a natural way to an increase of this width relative to the prediction of the standard model. Various strategies of a direct search for such CP–violating couplings by using CP–odd observables are outlined. The number of Z bosons required to obtain significant information on the couplings in this way is well within the reach of present LEP experiments.

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In a series of papers we have investigated possible new CP–violating couplings which may manifest themselves in Z boson decays, especially to τ leptons and to b quarks [1–6]. There is related work by other authors [7]. Experimental results on the CP–violating weak dipole moment of the τ lepton were published in [8]. In this note we want to update and extend the analyses presented in [1, 3] for the decays of the Z boson into bottom quarks. This seems timely because in recent years large samples of $b\bar{b}$ events have been selected by LEP experiments using microvertex detectors.

Important information on the strength of new couplings involving b quarks can be derived from the width $Z \to b\bar{b}X$. Its recent LEP average is [9]

$$R_b^{\rm exp} \equiv \frac{\Gamma(Z \to b\bar{b}X)}{\Gamma(Z \to {\rm hadrons})} = 0.2197 \pm 0.0020 \; . \tag{1}$$

This average includes the latest OPAL result [10] of $R_b = 0.2171 \pm 0.0021 \pm 0.0021$ obtained with a double tagging method. With a top quark mass of $m_t = 175 \pm 20$ GeV [11] the standard model (SM) value of R_b is [12]

$$R_b^{\rm SM} = 0.216 \pm 0.001 \; . \tag{2}$$

Hence there is a margin for new physics effects in $Z \to b\bar{b}X$.

Of course, even if $R_b^{\rm exp}$ agreed perfectly with the SM prediction new physics effects in $Z \to b\bar{b}X$ were not excluded. Such effects may cancel in the total decay rate $\Gamma(Z \to b\bar{b}X)$ but could show up in differential distributions. We will make some further comments on such a scenario below.

An interesting possibility is that the channels $Z \to b\bar{b}X$ are affected by CP–violating couplings such that the total amplitude $\mathcal{M}$ is the sum of the SM and a CP–violating one:

$$\mathcal{M} = \mathcal{M}_{\rm SM} + \mathcal{M}_{\rm CP} \; . \tag{3}$$

In this case the absolute squares of the amplitudes add incoherently in the width:

$$\Gamma(Z \to b\bar{b}X) \propto |\mathcal{M}_{\rm SM}|^2 + |\mathcal{M}_{\rm CP}|^2 \; , \tag{4}$$

thus leading naturally to an enhancement of the width. In this paper we discuss the magnitude of CP–violating effects one can expect in $\Gamma(Z \to b\bar{b}X)$ if indeed the small discrepancy between the experimental and the SM values for the width in (1), (2) is due to a CP–violating amplitude. Using [13]

$$\Gamma(Z \to {\rm hadrons}) = 1742 \ {\rm MeV} \; , \tag{5}$$

we obtain at the 1 s.d. level for a possible anomalous term $\Delta\Gamma$ in the width for $Z \to b\bar{b}X$:

$$\begin{aligned} \Delta\Gamma_{\rm exp}(Z \to b\bar{b}X) &\leq \Gamma_{\rm exp}(Z \to b\bar{b}X)\,|_{\rm mean\ value} - \Gamma_{\rm SM}(Z \to b\bar{b}X) \\ &\quad + \left[\delta\Gamma^2_{\rm exp}(1\ {\rm s.d.}) + \delta\Gamma^2_{\rm SM}(1\ {\rm s.d.})\right]^{1/2} \\ &= 0.0059 \cdot \Gamma(Z \to {\rm hadrons}) = 10.3\ {\rm MeV} \; . \end{aligned} \tag{6}$$

A systematic analysis of CP–violating couplings corresponding to operators with dimension ≤ 6 (after symmetry breaking) relevant for Z decays was given in eq. (4.1) of [1], where we wrote down the effective Lagrangian. For the decay $Z \to b\bar{b}X$ we have in this framework 7 CP–violating coupling constants: the electric, weak and chromoelectric dipole moments of the b (d_b, $\tilde{d}_b$, d'_b), the parameters f_{Vb}, f_{Ab} of the $Zbb\gamma$ vertex and the parameters h_{Vb}, h_{Ab} of the $ZbbG$ vertex. The dipole moments induce chirality–changing couplings and have dimension $(\mathrm{mass})^{-1}$. The couplings with f_{Vb}, f_{Ab}, h_{Vb}, h_{Ab} conserve chirality and these parameters have dimension $(\mathrm{mass})^{-2}$.

If we calculate the contribution to the width $\Gamma(Z \to b\bar{b}X)$ from the CP–odd couplings with these parameters $d_b, \ldots, h_{Ab}$, we get for $m_b \neq 0$ a rather lengthy expression containing also various interference terms. There, the contribution from the electric dipole moment of the b is proportional to $e^2|d_b|^2$ and thus of the same order as the electromagnetic radiative correction to the contribution from the weak dipole term $\tilde{d}_b$. Similarly, the contribution from the chromoelectric dipole moment d'_b is of order $e^2|d'_b|^2$ and we can give an argument that it should be discussed together with the QCD radiative correction to the weak dipole contribution. We plan to present all this in detail in a future publication.

In this paper we will adopt the following simple procedure: We will set the quark mass m_b to zero and neglect all terms of order e^2 times new couplings d_b, $\tilde{d}_b$, d'_b, f_{Vb}, f_{Ab}, h_{Vb}, h_{Ab} in the width. In this approximation the electric and chromoelectric dipole terms do not contribute. Then we calculate the contribution $\Delta\Gamma(Z \to b\bar{b}X)$ to the width assuming that either $\tilde{d}_b$ or f_{Vb}, f_{Ab} or h_{Vb}, h_{Ab} are different from zero. The result is shown in Table 1. Here and in the following we use the dimensionless parameters $\hat{f}_{Vb}$, $\hat{f}_{Ab}$, $\hat{h}_{Vb}$, $\hat{h}_{Ab}$ as defined in (5.1) of [1]:

$$\begin{aligned} f_{Vb/Ab} &= -\frac{e^2 Q_b}{\sin\vartheta_W \cos\vartheta_W m_Z^2} \hat{f}_{Vb/Ab} \, , \\ h_{Vb/Ab} &= \frac{e g_s}{\sin\vartheta_W \cos\vartheta_W m_Z^2} \hat{h}_{Vb/Ab} \, . \end{aligned} \tag{7}$$

The result for $\Delta\Gamma$ from the weak dipole term $\tilde{d}_b$ was already given in [1, 3]. In the numerical calculations we use for the fine structure constant at the Z mass α=1/129, m_Z=91.19 GeV, $\sin^2\vartheta_W$=0.23, α_s=0.12, and

$$\Gamma_{\nu_e\bar{\nu}_e} = \frac{\alpha m_Z}{24 \sin^2\vartheta_W \cos^2\vartheta_W O = 166 \text{ MeV} \, .} \tag{8}$$

From Table 1 we deduce limits on our CP–violating couplings using as input $\Delta\Gamma_{\mathrm{exp}}$ from (6):

$$\begin{aligned} |\tilde{d}_b| &\leq 3.8 \cdot 10^{-17} \mathrm{e\,cm} \, , \\ \left[(\hat{f}_{Vb})^2 + (\hat{f}_{Ab})^2\right]^{1/2} &\leq 27.5 \, , \\ \left[(\hat{h}_{Vb})^2 + (\hat{h}_{Ab})^2\right]^{1/2} &\leq 2.0 \, . \end{aligned} \tag{9}$$

We see that the measured width for $Z \to b\bar{b}X$ allows in principle quite sizeable CP–violating couplings for the b quarks. In Z production at electron–positron colliders such couplings can be searched for directly using the methods of [1–6]. Here we add a few comments and present some new calculations.

With unpolarized e^- and e^+ beams the initial state is CP–symmetric, which is the case relevant for LEP. At SLC with polarized electrons and unpolarized positrons the initial state is not CP–symmetric. Nevertheless, in leading order one can consider the reaction as a two-step process

$$e^+e^- \longrightarrow Z \longrightarrow b\bar{b}X \; . \tag{10}$$

The Z in its rest system is a CP eigenstate (cf. [1]). To leading order CP–violating correlations in the final state are then also an indicator of CP violation in Z decays. Some calculations for this case were recently presented in [14]. But now radiative corrections for polarized e^+e^- collisions have to be carefully examined for their possibility of faking CP violation. In the following we will, therefore, concentrate on the case of e^+e^- collisions with unpolarized beams.

In the decay of the Z boson into a pair of b quarks fragmenting subsequently into B hadrons various final states can occur: two jets, three jets, two jets plus photon etc. We shall now discuss these cases with respect to CP violation.

The angular distribution of the jets in two jet decays of the Z does not carry any information on CP violation, as shown in [1]. To do useful CP studies with the decay $Z \to b\bar{b} \to 2$ jets one has to analyse b–$\bar{b}$ spin and/or spin–momentum correlations. Thus one needs a spin analyzer for b and $\bar{b}$. Parity–odd correlations of the fragmentation products of the b, $\bar{b}$ (the"handedness" of the b and $\bar{b}$ jets) may provide such spin analyzers [15–18]. But the relevant analyses have just started to be done [19]. Thus, at the moment we cannot obtain any information on CP–violating couplings from 2 jet events.

We turn to decays of the Z into 3 jets and 2 jets plus one photon. At the parton level this means

$$\begin{aligned} Z &\to b\bar{b}G \; , & (11)\\ Z &\to b\bar{b}\gamma \; . & (12)\end{aligned}$$

We have studied these decays extensively in [1]. In this note we present some further calculations of CP–odd quantities for (11), (12) and estimates of the number of events which are necessary to detect CP–violating couplings of a magnitude given by (9). We will describe now various analysis scenarios.

I Analysis of $Z \to$ 3 jets, flavour blind case

Consider the decay $Z \to$ 3 jets (11):

$$Z \to \mathrm{jet}(k_1) + \mathrm{jet}(k_2) + \mathrm{jet}(k_3) \; . \tag{13}$$

Let us assume that one can select events tagged by at least one B decay and that the jets are ordered according to the magnitude of their momenta (cf. (3.26a) of [1]):

$$|\mathbf{k}_1| \geq |\mathbf{k}_2| \geq |\mathbf{k}_3| \ . \tag{14}$$

This the "flavour blind" case considered in [1], where we showed that in the limit $m_b = 0$ only the couplings $\hat{h}_{Vb}$, $\hat{h}_{Ab}$ can induce CP–violating effects in 3 jet decays. We found furthermore that all such CP–odd effects are then proportional to the following linear combination of $\hat{h}_{Vb}$, $\hat{h}_{Ab}$:

$$\hat{h}_b := \hat{h}_{Ab} g_{Vb} - \hat{h}_{Vb} g_{Ab} \ , \tag{15}$$

where g_{Vb}, g_{Ab} are the SM vector and axial vector couplings of the b quark to the Z boson:

$$g_{Vb} = -\frac{1}{2} + \frac{2}{3}\sin^2\vartheta_W \ , \quad g_{Ab} = -\frac{1}{2} \ . \tag{16}$$

From the limit on $\hat{h}_{Vb}^2 + \hat{h}_{Ab}^2$ given in (9) we get

$$|\hat{h}_b| \leq 1.2 \ . \tag{17}$$

Assuming now that only $\hat{h}_{Vb}$, $\hat{h}_{Ab}$ are nonzero we calculate the expectation values and the variances of the following CP–odd observables (cf. (3.39) of [1] and (2.20) of [3]):

$$T_{ij}^{(a)} = \hat{k}_{ai}\hat{n}_j + \hat{k}_{aj}\hat{n}_i \quad (a = 1,2,3) \ . \tag{18}$$

Here i, j ($1 \leq i, j \leq 3$) are the Cartesian vector indices (with 3 being the positron beam direction) and

$$\hat{\mathbf{k}}_a = \frac{\mathbf{k}_a}{|\mathbf{k}_a|} \ , \quad \hat{\mathbf{n}} = \frac{\hat{\mathbf{k}}_1 \times \hat{\mathbf{k}}_2}{|\hat{\mathbf{k}}_1 \times \hat{\mathbf{k}}_2|} \ . \tag{19}$$

We use a y-cut to define our sample of 3 jet events at parton level, i. e. we require

$$\frac{(k_a + k_b)^2}{m_Z^2} \geq y_{\mathrm{cut}} \ , \quad (1 \leq a \neq b \leq 3) \ , \tag{20}$$

where y_{cut} is a conveniently chosen parameter. For a given y_{cut} we can expand the width and the expectation values of the correlations $T_{33}^{(a)}$ as follows:

$$\Gamma(Z \to b\bar{b}G) = \Gamma_{b\bar{b}G}^{\mathrm{SM}} + \left[(\hat{h}_{Vb})^2 + (\hat{h}_{Ab})^2\right]\Gamma'_{b\bar{b}G} \ , \tag{21}$$

$$\langle T_{33}^{(a)} \rangle \Gamma(Z \to b\bar{b}G) = \Gamma_{b\bar{b}G}^{\mathrm{SM}} Y^{(a)} \hat{h}_b \quad (a = 1,2,3) \ . \tag{22}$$

Numerical results for $\Gamma_{b\bar{b}G}^{\mathrm{SM}}$, $\Gamma'_{b\bar{b}G}$ and $Y^{(a)}$ are presented in Tables 2,3 for y_{cut}=0.03, 0.05 and 0.10. We also list the values for the variances $\langle (T_{33}^{(a)})^2 \rangle$ as calculated with the SM amplitude alone. With this information we can calculate the number N_{cut} of events $Z \to b\bar{b}G$ within the corresponding y-cuts which are needed in order to see a nonzero expectation value for $T_{33}^{(a)}$ with a significance of 1 s. d., given a value for $\hat{h}_b$. We also list the corresponding total number N_{tot} of Z bosons needed. We have for a measurement of $T_{33}^{(a)}$:

$$
\begin{aligned}
N_{\rm cut} &= \frac{1}{|\hat{h}_b|^2}\frac{\langle (T_{33}^{(a)})^2\rangle}{|Y_a|^2}\ , \\
N_{\rm tot} &= N_{\rm cut}\frac{\Gamma_Z}{\Gamma^{\rm SM}_{b\bar{b}G}}\ .
\end{aligned}
\tag{23}
$$

Here Γ_Z is the total Z width and we set any non-standard contribution to the widths to zero. For the numerics we use $\Gamma_Z = 2497$ MeV [20]. We see from Table 3 that for $|\hat{h}_b| = 1$ – which is perfectly allowed by the experimental results for R_b (cf. (17)) – the number of Z bosons required to see CP–odd effects is not outside the reach of today's experiments. Note, however, that in calculating $N_{\rm tot}$ (23) we have assumed all experimental efficiencies, for B-tagging etc., to be equal to one. In real life, efficiencies less than one are unavoidable and will increase the number of Z bosons required to see CP–odd effects of a given magnitude. This remark applies as well to all numbers $N_{\rm tot}$ given below for other observables.

Now we turn to "optimal" observables [21–23]. Let us assume that only the distribution of the unit momenta $\hat{\mathbf{k}}_a$ (a=1,2,3) of the three jets is analysed. Then we write the decay distribution of the 3 jets in the reaction $e^+e^- \to Z \to b\bar{b}G \to 3$ jets in the following way (cf. (6) and (31) of [22]):

$$
\begin{aligned}
&\frac{1}{\Gamma(Z\to b\bar{b}G)} d\Gamma(Z\to b\bar{b}G \to 3\ \text{jets}) = \\
&\qquad = \frac{1}{\Gamma^{\rm SM}_{b\bar{b}G}}\Big(S_0 + S_1\hat{h}_b + \text{terms quadratic in } \hat{h}_{Vb}, \hat{h}_{Ab}\Big) d\phi\ .
\end{aligned}
\tag{24}
$$

Here

$$
d\phi = \delta[(\hat{\mathbf{k}}_1 \times \hat{\mathbf{k}}_2)\cdot\hat{\mathbf{k}}_3] d\Omega_1 d\Omega_2 d\Omega_3 \Theta(|\mathbf{k}_1| - |\mathbf{k}_2|)\Theta(|\mathbf{k}_2| - |\mathbf{k}_3|) \tag{25}
$$

is the phase space measure with $d\Omega_a$ the solid angle element to $\hat{\mathbf{k}}_a$ (a=1,2,3). The δ-function in (25) takes into account that a three body final state is planar. A suitable observable to measure $\hat{h}_b$ is then

$$
O = \frac{S_1}{S_0}\ . \tag{26}
$$

This observable is the optimal one if in (24) quadratic terms in $\hat{h}_{Vb}$, $\hat{h}_{Ab}$ are negligible. The explicit form of S_0, S_1 is easily derived from the formulae given in [1]. We obtain:

$$
\begin{aligned}
S_0 &= (g_{Vb}^2 + g_{Ab}^2)\frac{24\alpha_s\Gamma_{\nu_e\bar{\nu}_e}}{\pi^3}\frac{x_1x_2x_3}{y_1y_2y_3}\left\{\Big[x_1^3\Big(1 + (\hat{\mathbf{p}}_+\cdot\hat{\mathbf{k}}_1)^2\Big)\Big] + \begin{matrix}\text{cycl. perm.'s}\\ \text{in (1,2,3)}\end{matrix}\right\}, \\
S_1 &= \frac{24\alpha_s\Gamma_{\nu_e\bar{\nu}_e}}{\pi^3}x_1x_2x_3\ \times \\
&\qquad \times\left\{\Big[x_1^2x_2\Big(\frac{1}{y_3} - \frac{1}{y_2}\Big)\,\hat{\mathbf{p}}_+\cdot(\hat{\mathbf{k}}_1\times\hat{\mathbf{k}}_2)\,\hat{\mathbf{p}}_+\cdot\hat{\mathbf{k}}_1\Big] + \begin{matrix}\text{cycl. perm.'s}\\ \text{in (1,2,3)}\end{matrix}\right\}.
\end{aligned}
\tag{27}
$$

Here $\hat{\mathbf{p}}_+$ is the unit vector in the direction of the e^+ beam and

$$x_a = \frac{k_{a0}}{m_Z} , \quad y_a = 1 - 2x_a \quad (a = 1, 2, 3) . \tag{28}$$

We have written O in (26), (27) in such a way that it is a perfect CP–odd observable also for 3 jet decays of the Z as observed experimentally where the 3 jets are in general not exactly planar due to photon radiation in the initial and/or final state, jet reconstruction errors etc. For the expectation value we write

$$\langle O \rangle \Gamma(Z \to b\bar{b}G) = \Gamma^{\rm SM}_{b\bar{b}G} C \hat{h}_b . \tag{29}$$

In Table 4 we present the results of our numerical calculations for C for three values of $y_{\rm cut}$. The quantity C also represents the expectation value of O^2 in the SM (cf. [22]):

$$\langle O^2 \rangle_{\rm SM} = \frac{\int S_0 (S_1/S_0)^2 d\phi}{\int S_0 d\phi} = C . \tag{30}$$

For the numbers $N_{\rm cut}$ and $N_{\rm tot}$ required to see an effect at the 1 s. d. level we have here

$$\begin{aligned} N_{\rm cut} &= \frac{1}{|\hat{h}_b|^2} \frac{1}{C} , \\ N_{\rm tot} &= N_{\rm cut} \frac{\Gamma_Z}{\Gamma^{\rm SM}_{b\bar{b}G}} . \end{aligned} \tag{31}$$

These numbers are also listed in Table 4. Comparing the results of Tables 3 and 4 we find that the optimal observable (29) leads to some but not a dramatic gain in sensitivity compared to the observables $T^{(a)}_{33}$.

II Analysis of $Z \to$ 3 jets, identification of the highest energy jet as coming from b or $\bar{b}$ fragmentation

Here we consider the following type of analysis: In the decay $Z \to 3$ jets at least one B hadron is observed. When the three jets are ordered according to (13), (14) one requires that the jet 1 which has the highest absolute value of momentum contains a B hadron. Let us assume that after this selection of events the further analysis proceeds as in I with the jet ordering criterion (14).

No we discuss the implications of the selection of events as described above at parton level. If we order the jets in $Z \to b\bar{b}G$ according to (14) we can distinguish the 6 classes of events shown in Table 5. With the procedure described above we select only the events corresponding to the first 4 classes in Table 5.

The further analysis follows the same lines as in I. In Table 6 we list $\Gamma^{\rm SM}_{b\bar{b}G,{\rm II}}$ and $\Gamma'_{b\bar{b}G,{\rm II}}$ defined as in (21) but with the selection II imposed. Comparing the results of Tables 2 and 6 we see that the 4 subclasses account for over 90 % of the events, i.e. it is very unlikely to have a gluon jet as most energetic jet. In Table 7 we list the

values for parameters related to the CP–odd observables $T_{33}^{(a)}$ $(a = 1,2,3)$ of (18) again with the selection II imposed:

$$\langle T_{33}^{(a)} \rangle_{\rm II} \Gamma_{\rm II}(Z \to b\bar{b}G) = \Gamma_{b\bar{b}G,{\rm II}}^{\rm SM} Y_{\rm II}^{(a)} \hat{h}_b \ . \tag{32}$$

We discuss now the optimal observable for analysis II. We have here

$$\begin{aligned} \frac{1}{\Gamma_{\rm II}(Z \to b\bar{b}G)} d\Gamma_{\rm II}(Z \to b\bar{b}G \to 3 \text{ jets}) \ = \\ = \ \frac{1}{\Gamma_{b\bar{b}G,{\rm II}}^{\rm SM}} \Big(\tilde{S}_0 + \tilde{S}_1 \hat{h}_b + \text{terms quadratic in } \hat{h}_{Vb}, \hat{h}_{Ab} \Big) d\phi \ , \end{aligned} \tag{33}$$

where

$$\begin{aligned} \tilde{S}_0 \ &= \ (g_{Vb}^2 + g_{Ab}^2) \frac{12\alpha_s \Gamma_{\nu_e\bar{\nu}_e}}{\pi^3} x_1 x_2 x_3 \Big\{ \Big[\frac{1}{y_1 y_2} \Big(x_1^2 \left(1 + (\hat{\mathbf{p}}_+ \cdot \hat{\mathbf{k}}_1)^2\right) + \\ &\qquad + x_2^2 \left(1 + (\hat{\mathbf{p}}_+ \cdot \hat{\mathbf{k}}_2)^2\right) \Big) \Big] + [2 \leftrightarrow 3] \Big\} , \\ \tilde{S}_1 \ &= \ \frac{24\alpha_s \Gamma_{\nu_e\bar{\nu}_e}}{\pi^3} x_1 x_2 x_3 \ \times \\ &\qquad \times \Big\{ \Big[x_1 x_2 \, \hat{\mathbf{p}}_+ \cdot (\hat{\mathbf{k}}_1 \times \hat{\mathbf{k}}_2) \Big(\frac{x_2}{y_1} \hat{\mathbf{p}}_+ \cdot \hat{\mathbf{k}}_2 - \frac{x_1}{y_2} \hat{\mathbf{p}}_+ \cdot \hat{\mathbf{k}}_1 \Big) \Big] + [2 \leftrightarrow 3] \ \Big\} \ . \end{aligned} \tag{34}$$

The optimal observable is then

$$O_{\rm II} = \frac{\tilde{S}_1}{\tilde{S}_0} \ , \tag{35}$$

and we write for its expectation value

$$\langle O_{\rm II} \rangle \Gamma_{\rm II}(Z \to b\bar{b}G) = \Gamma_{b\bar{b}G,{\rm II}}^{\rm SM} \, C_{\rm II} \, \hat{h}_b \ . \tag{36}$$

Values for $C_{\rm II}$ and $N_{\rm cut}$, $N_{\rm tot}$ defined in analogy to (31) are listed in Table 8.

III Analysis of $Z \to 3$ jets, identification of the second highest energy jet as coming from b or $\bar{b}$ fragmentation

Again we consider decays $Z \to 3$ jets where at least one B hadron is observed, with the jet ordering (14). Contrary to the analysis II we now demand a B tag in jet 2, which is the jet with the second highest energy. Looking through Table 5 we see that this corresponds to the event classes 1,2 and 5,6. Note that this selection III is not complementary to the selection II above; events where the least energetic jet comes from the fragmentation of a gluon are used both for analysis II and III.

In Table 9 we give the values $\Gamma_{b\bar{b}G,{\rm III}}^{\rm SM}$ and $\Gamma'_{b\bar{b}G,{\rm III}}$ (cf. (21)) for the selection III. The definition of the numbers $Y_{\rm III}^{(a)}$ characterizing the expectation values of the CP–odd observable $T_{33}^{(a)}$ in the event sample III is identical to (32),

$$\langle T_{33}^{(a)} \rangle_{\rm III} \Gamma_{\rm III}(Z \to b\bar{b}G) = \Gamma_{b\bar{b}G,{\rm III}}^{\rm SM} Y_{\rm III}^{(a)} \hat{h}_b \ , \tag{37}$$

and we list the relevant results in Table 10.

The optimal observable for the present analysis is

$$O_{\mathrm{III}} = \frac{\tilde{S}_1(1 \leftrightarrow 2)}{\tilde{S}_0(1 \leftrightarrow 2)} \;, \tag{38}$$

where $\tilde{S}_0(1 \leftrightarrow 2)$, $\tilde{S}_1(1 \leftrightarrow 2)$ are obtained from (34) by exchanging the indices $(1 \leftrightarrow 2)$. The expectation value $\langle O_{\mathrm{III}} \rangle$ defines the numbers C_{III}:

$$\langle O_{\mathrm{III}} \rangle \Gamma_{\mathrm{III}}(Z \to b\bar{b}G) = \Gamma^{\mathrm{SM}}_{b\bar{b}G,\mathrm{III}}\, C_{\mathrm{III}}\, \hat{h}_b \;. \tag{39}$$

In Table 11 we present values for C_{III} and N_{cut}, N_{tot} for various values of y_{cut}. It is clear why the analysis III has a higher sensitivity compared to analysis II: As noted above, it is very unlikely that jet 1 is a gluon jet. If one requires jet 2 to come from the fragmentation of a b ($\bar{b}$), one has then with a high probability the full information of the parton content of the jets.

IV Analysis of $Z \to$ 3 jets with flavour identification

Here we discuss the case where two of the three jets in $Z \to 3$ jets have been tagged by the observation of a B hadron as coming from the fragmentation of a b or $\bar{b}$ quark. Knowledge of the charge of the parent quark of the jet is not required. Let $\hat{\mathbf{k}}_+$ ($\hat{\mathbf{k}}_-$) be the momentum direction of the $\bar{b}$ (b) jet. Then we can form the following CP–odd tensor observable:

$$T'_{ij} = (\hat{\mathbf{k}}_+ - \hat{\mathbf{k}}_-)_i \left(\frac{\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_-}{|\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_-|} \right)_j + (i \leftrightarrow j) \;, \tag{40}$$

where $1 \le i, j \le 3$. Note that T'_{ij} is invariant under $\hat{\mathbf{k}}_+ \leftrightarrow \hat{\mathbf{k}}_-$. Thus the correct and the wrong assignment of the momenta $\hat{\mathbf{k}}_\pm$ to the b and $\bar{b}$ jets give the same result for T'_{ij}. As above we compute

$$\langle T'_{33} \rangle \Gamma(Z \to b\bar{b}G) = \Gamma^{\mathrm{SM}}_{b\bar{b}G} Y' \hat{h}_b \;. \tag{41}$$

The quantities $\Gamma(Z \to b\bar{b}G)$ and $\Gamma^{\mathrm{SM}}_{b\bar{b}G}$ for given y_{cut} are here identical to the ones defined in (21) for analysis I. The corresponding numerical results are given in Table 2. Numerical results for Y' and some other parameters related to the observable T'_{33} are collected in Table 12. The numbers N_{tot} required to see possible CP–odd effects at the 1 s.d. level are surprisingly low here. We have, however, to remember that in our calculations all efficiencies were assumed to be equal to one. In reality, the double B-tag efficiency is considerably smaller than the efficiency for a single B-tag. Thus the gain in sensitivity which is in principle obtainable by the double tag method IV may in practice be partly or completely lost due to the smaller efficiency.

V Analysis of $Z \to$ 2 jets + γ

Here we discuss the decay (12):

$$Z \to b\bar{b}\gamma \to \mathrm{jet}(k_+) + \mathrm{jet}(k_-) + \gamma(k) \ . \tag{42}$$

Experimentally this class of events can be selected by requiring two jets, at least one B hadron tag and one isolated photon. To define the event sample we use again the y-cut (20) applied to the two jets and the photon as third "jet". We write then for the width similarly to (21):

$$\Gamma(Z \to b\bar{b}\gamma) = \Gamma^{\mathrm{SM}}_{b\bar{b}\gamma} + \left[(\hat{f}_{Vb})^2 + (\hat{f}_{Ab})^2\right]\Gamma'_{b\bar{b}\gamma} \ . \tag{43}$$

Numerical results for $\Gamma^{\mathrm{SM}}_{b\bar{b}\gamma}$ and $\Gamma'_{b\bar{b}\gamma}$ are shown in Table 13.

As CP–odd observable we can choose here again T'_{ij} defined in (40) where the assignment of the momenta k_+ and k_- to the b and $\bar{b}$ jet or vice versa does not matter. The expectation values of T'_{ij} depend only on $\hat{f}_b$ defined as

$$\hat{f}_b := \hat{f}_{Ab} g_{Vb} - \hat{f}_{Vb} g_{Ab} \ . \tag{44}$$

This can be seen from the formulae in Appendix B of [1]. The restriction on $(\hat{f}_{Vb})^2 + (\hat{f}_{Ab})^2$ in (9) leads to

$$|\hat{f}_b| \leq 16.7 \ . \tag{45}$$

For the expectation value of T'_{33} we get here:

$$\langle T'_{33}\rangle \Gamma(Z \to b\bar{b}\gamma) = \Gamma^{\mathrm{SM}}_{b\bar{b}\gamma} Y'_\gamma \hat{f}_b \ . \tag{46}$$

Numerical results for Y'_γ and for $\langle (T'_{33})^2\rangle$, N_{cut} and N_{tot} defined as

$$\begin{aligned} N_{\mathrm{cut}} &= \frac{1}{|\hat{f}_b|^2} \frac{\langle (T'_{33})^2\rangle}{|Y'_\gamma|^2} \ , \\ N_{\mathrm{tot}} &= N_{\mathrm{cut}} \frac{\Gamma_Z}{\Gamma^{\mathrm{SM}}_{b\bar{b}\gamma}} \end{aligned} \tag{47}$$

are shown in Table 14. Here the same procedure for the calculation as explained after (22) was followed.

The decay $Z \to b\bar{b}\gamma$ was studied with respect to CP–violating effects also by Abraham and Lampe [7]. They used various CP–odd asymmetries.

Finally we make some concluding remarks. In this paper we have studied the hypothesis that CP–violating couplings are responsible for a small increase in the width $\Gamma(Z \to b\bar{b}X)$ compared to the SM prediction. We have shown that this can be checked by direct searches for CP violation using appropriate CP–odd observables. The number of Z bosons required to obtain significant information on the relevant parameters of CP violation is well within the reach of present LEP experiments. We should emphasize here that the bounds (9) obtained for the CP–violating parameters

from the width $\Gamma(Z \to b\bar{b}X)$ depend crucially on the ansatz (3) for the amplitude $\mathcal{M}$. If new couplings play a role in the decays $Z \to b\bar{b}X$ one would in general expect that they contribute both to the CP–conserving and the CP–violating part of the amplitude $\mathcal{M}$. Then the CP–conserving new couplings may reduce the width. This can happen for instance in extensions of the SM with several Higgs doublets [24]. For such models the width measurement, i.e. the ratio (1) yields a margin for CP–violating couplings which is bigger than the bounds (9) suggest. Clearly, the width measurement is no substitute for a direct search for CP violation. In [1] we have, in fact, given a complete list of CP relations which can be checked in three body decays of the Z (cf. Table 1 and (3.12), (3.28) of [1]). In the present paper we have considered tensor and optimal observables. The latter turn out to have very small variances in theory. In real life, resolution effects, measurement errors etc. will certainly increase the variances and one has to check in each case if these observables are then still better than e.g. the T_{33} variables. Looking through the numbers of the Tables we see that the most promising way to search for CP–violating effects is open if two jets can be tagged as coming from a b or $\bar{b}$ quark (analyses IV, V). If also the charge of the jets can be determined – for instance by observing the charge of the lepton in a semileptonic B decay – one has further interesting CP–odd observables at one's disposal (cf. [1]).

In our opinion a study of CP–violating couplings in Z decays to final states containing B hadrons deserves attention. We emphasize again that we are concerned here not with CP violation in B hadron decays but with CP violation in the $Zb\bar{b}$, $Zb\bar{b}G$ and $Zb\bar{b}\gamma$ vertices. We encourage experimentalists to explore this field.

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Table Captions

Table 1: Contribution to the width for $Z \to b\bar{b}X$ from different anomalous CP–violating couplings. The first column lists the coupling parameter, the second column the final state to which the coupling contributes. In the third column we give the results for $\Delta\Gamma$ where the b quark mass has been set to zero.

Table 2: The SM width for $Z \to b\bar{b}G$ and the width $\Gamma'_{b\bar{b}G}$ as defined in (21) for three values of $y_{\rm cut}$ (20).

Table 3: Values for parameters related to the CP–odd observables $T_{33}^{(a)}$ (a=1,2,3) defined in (18). Listed are $y_{\rm cut}$, $Y^{(a)}$, $\langle (T_{33}^{(a)})^2 \rangle$, $|\hat{h}_b|^2 N_{\rm cut}$ and $|\hat{h}_b|^2 N_{\rm tot}$ (cf. (20), (22), (23)).

Table 4: Values for parameters related to the CP–odd optimal observable O defined in (26), (27). Listed are $y_{\rm cut}$ (20), C (29), (30), $N_{\rm cut}$ and $N_{\rm tot}$ (31).

Table 5: The 6 possibilities for b, $\bar{b}$ and G to give 3 jets with momenta $|\mathbf{k}_1| \geq |\mathbf{k}_2| \geq |\mathbf{k}_3|$. Only the events corresponding to the first 4 rows satisfy the selection criterion II.

Table 6: The SM width for $Z \to b\bar{b}G$ and the width $\Gamma'_{b\bar{b}G,{\rm II}}$ as defined in (21) for three values of $y_{\rm cut}$ (20) and selection II imposed.

Table 7: Values for parameters related to the CP–odd observables $T_{33}^{(a)}$ (a=1,2,3) defined in (18) but now with the selection II imposed. Listed are $y_{\rm cut}$, $Y^{(a)}$, $\langle (T_{33}^{(a)})^2 \rangle$, $|\hat{h}_b|^2 N_{\rm cut}$ and $|\hat{h}_b|^2 N_{\rm tot}$ (cf. (20), (22), (23)).

Table 8: Values for parameters related to the CP–odd optimal observable $O_{\rm II}$ relevant if the selection II is applied (cf. (33)–(36)).

Table 9: The SM width $\Gamma^{\rm SM}_{b\bar{b}G,{\rm III}}$ and the width $\Gamma'_{b\bar{b}G,{\rm III}}$ (cf. (21)) for selection III.

Table 10: Values for parameters related to the CP–odd observables $T_{33}^{(a)}$ (a=1,2,3) for analysis III. Listed are $y_{\rm cut}$, $Y^{(a)}$, $\langle (T_{33}^{(a)})^2 \rangle$, $|\hat{h}_b|^2 N_{\rm cut}$ and $|\hat{h}_b|^2 N_{\rm tot}$.

Table 11: $C_{\rm III}$, $|\hat{h}_b|^2 N_{\rm cut}$ and $|\hat{h}_b|^2 N_{\rm tot}$ for the optimal observable $O_{\rm III}$ of analysis III (cf. (38), (39)) for three values of $y_{\rm cut}$.

Table 12: Values for parameters related to the CP–odd observable T'_{33} (40). Listed are $y_{\rm cut}$, Y', $\langle (T'_{33})^2 \rangle$, $|\hat{h}_b|^2 N_{\rm cut}$, $|\hat{h}_b|^2 N_{\rm tot}$.

Table 13: Numerical results for $\Gamma^{\rm SM}_{b\bar{b}\gamma}$ and $\Gamma'_{b\bar{b}\gamma}$ as defined in (43).

Table 14: Values for parameters related to the CP–odd observable T'_{33} defined in (40) but applied to the decay $Z \to b\bar{b}\gamma$ (42). Listed are Y'_γ, $\langle (T'_{33})^2 \rangle$, $|\hat{f}_b|^2 N_{\rm cut}$ and $|\hat{f}_b|^2 N_{\rm tot}$ for three values of $y_{\rm cut}$ (cf. (46), (47), (20)).

Table 1

coupling parameter	final state	$\Delta\Gamma(Z \to b\bar{b}X)$
$\tilde{d}_b$	$b\bar{b}$	$\lvert\tilde{d}_b\rvert^2 \frac{m_Z^3}{8\pi} = $ $= (\lvert\tilde{d}_b\rvert \cdot 10^{17}\mathrm{e}^{-1}\mathrm{cm}^{-1})^2 \cdot 0.71$ MeV
$\hat{f}_{Vb},\ \hat{f}_{Ab}$	$b\bar{b}\gamma$	$\left[(\hat{f}_{Vb})^2 + (\hat{f}_{Ab})^2\right] \frac{\alpha}{30\pi} \Gamma_{\nu_e\bar{\nu}_e} = $ $= \left[(\hat{f}_{Vb})^2 + (\hat{f}_{Ab})^2\right] \cdot 1.4 \cdot 10^{-2}$ MeV
$\hat{h}_{Vb},\ \hat{h}_{Ab}$	$b\bar{b}G$	$\left[(\hat{h}_{Vb})^2 + (\hat{h}_{Ab})^2\right] \frac{2\alpha_s}{5\pi} \Gamma_{\nu_e\bar{\nu}_e} = $ $= \left[(\hat{h}_{Vb})^2 + (\hat{h}_{Ab})^2\right] \cdot 2.54$ MeV

Table 2

$y_{\rm cut}$	$\Gamma^{\rm SM}_{b\bar{b}G}$ [MeV]	$\Gamma'_{b\bar{b}G}$ [MeV]
0.03	127.82	2.214
0.05	81.33	1.990
0.10	34.69	1.437

Table 3

$y_{\rm cut}$	$Y^{(1)}$	$\langle (T_{33}^{(1)})^2 \rangle$	$\lvert \hat{h}_b \rvert^2\, N_{\rm cut}$	$\lvert \hat{h}_b \rvert^2\, N_{\rm tot}$
0.03	–0.00364	0.282	21 316	415 712
0.05	–0.00452	0.281	13 731	421 736
0.10	–0.00567	0.277	8 595	618 793
$y_{\rm cut}$	$Y^{(2)}$	$\langle (T_{33}^{(2)})^2 \rangle$	$\lvert \hat{h}_b \rvert^2\, N_{\rm cut}$	$\lvert \hat{h}_b \rvert^2\, N_{\rm tot}$
0.03	0.00445	0.280	14 083	274 654
0.05	0.00565	0.277	8 677	266 511
0.10	0.00737	0.272	5 002	360 148
$y_{\rm cut}$	$Y^{(3)}$	$\langle (T_{33}^{(3)})^2 \rangle$	$\lvert \hat{h}_b \rvert^2\, N_{\rm cut}$	$\lvert \hat{h}_b \rvert^2\, N_{\rm tot}$
0.03	0.00010	0.246	24 829 426	484 221 122
0.05	–0.00026	0.244	3 670 817	112 740 730
0.10	–0.00110	0.243	199 527	14 363 713

Table 4

$y_{\rm cut}$	C	$\lvert \hat{h}_b \rvert^2\, N_{\rm cut}$	$\lvert \hat{h}_b \rvert^2\, N_{\rm tot}$
0.03	0.0001113	8 988	175 332
0.05	0.0001566	6 385	196 102
0.10	0.0002382	4 197	302 314

Table 5

jet 1	jet 2	jet 3
b	$\bar{b}$	G
$\bar{b}$	b	G
b	G	$\bar{b}$
$\bar{b}$	G	b
G	b	$\bar{b}$
G	$\bar{b}$	b

Table 6

y_{cut}	$\Gamma^{\mathrm{SM}}_{b\bar{b}G,\mathrm{II}}$ [MeV]	$\Gamma'_{b\bar{b}G,\mathrm{II}}$ [MeV]
0.03	121.35	1.033
0.05	75.63	0.963
0.10	30.75	0.751

Table 7

y_{cut}	$Y^{(1)}_{\mathrm{II}}$	$\langle (T^{(1)}_{33})^2 \rangle_{\mathrm{II}}$	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{cut}}$	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{tot}}$
0.03	–0.00402	0.283	17 543	360 762
0.05	–0.00514	0.282	10 660	352 016
0.10	–0.00696	0.279	5 757	467 638
y_{cut}	$Y^{(2)}_{\mathrm{II}}$	$\langle (T^{(2)}_{33})^2 \rangle_{\mathrm{II}}$	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{cut}}$	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{tot}}$
0.03	0.00568	0.280	8 668	178 259
0.05	0.00763	0.278	4 769	157 471
0.10	0.01168	0.272	1 995	162 099
y_{cut}	$Y^{(3)}_{\mathrm{II}}$	$\langle (T^{(3)}_{33})^2 \rangle_{\mathrm{II}}$	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{cut}}$	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{tot}}$
0.03	–0.00106	0.246	218 297	4 488 952
0.05	–0.00210	0.244	55 367	1 828 200
0.10	–0.00509	0.243	9 346	759 182

Table 8

y_{cut}	C_{II}	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{cut}}$	$\lvert\hat{h}_b\rvert^2\, N_{\mathrm{tot}}$
0.03	0.000244	4 106	84 494
0.05	0.000368	2 714	89 552
0.10	0.000717	1 394	113 252

Table 9

$y_{\rm cut}$	$\Gamma^{\rm SM}_{b\bar{b}G,{\rm III}}$ [MeV]	$\Gamma'_{b\bar{b}G,{\rm III}}$ [MeV]
0.03	105.16	1.552
0.05	65.07	1.378
0.10	26.41	0.971

Table 10

$y_{\rm cut}$	$Y^{(1)}_{\rm III}$	$\langle (T^{(1)}_{33})^2 \rangle_{\rm III}$	$\vert\hat{h}_b\vert^2 \, N_{\rm cut}$	$\vert\hat{h}_b\vert^2 \, N_{\rm tot}$
0.03	–0.00799	0.282	4 426	105 148
0.05	–0.01049	0.281	2 552	97 961
0.10	–0.01532	0.277	1 180	111 609
$y_{\rm cut}$	$Y^{(2)}_{\rm III}$	$\langle (T^{(2)}_{33})^2 \rangle_{\rm III}$	$\vert\hat{h}_b\vert^2 \, N_{\rm cut}$	$\vert\hat{h}_b\vert^2 \, N_{\rm tot}$
0.03	0.00730	0.282	5 287	125 614
0.05	0.00935	0.280	3 200	122 848
0.10	0.01257	0.276	1 747	165 246
$y_{\rm cut}$	$Y^{(3)}_{\rm III}$	$\langle (T^{(3)}_{33})^2 \rangle_{\rm III}$	$\vert\hat{h}_b\vert^2 \, N_{\rm cut}$	$\vert\hat{h}_b\vert^2 \, N_{\rm tot}$
0.03	0.00425	0.243	13 445	319 414
0.05	0.00533	0.241	8 478	325 438
0.10	0.00766	0.239	4 068	384 834

Table 11

$y_{\rm cut}$	$C_{\rm III}$	$\vert\hat{h}_b\vert^2 \, N_{\rm cut}$	$\vert\hat{h}_b\vert^2 \, N_{\rm tot}$
0.03	0.000377	2 650	62 957
0.05	0.000553	1 807	69 450
0.10	0.000985	1 015	96 016

Table 12

$y_{\rm cut}$	Y'	$\langle (T'_{33})^2 \rangle$	$\lvert\hat{h}_b\rvert^2\, N_{\rm cut}$	$\lvert\hat{h}_b\rvert^2\, N_{\rm tot}$
0.03	–0.02202	1.040	2 146	41 893
0.05	–0.02910	1.009	1 191	36 577
0.10	–0.04400	0.947	489	35 216

Table 13

$y_{\rm cut}$	$\Gamma^{\rm SM}_{b\bar{b}\gamma}$ [keV]	$\Gamma'_{b\bar{b}\gamma}$ [keV]
0.03	688.8	11.92
0.05	438.0	10.72
0.10	186.7	7.73

Table 14

$y_{\rm cut}$	Y'_γ	$\langle (T'_{33})^2 \rangle$	$\lvert\hat{f}_b\rvert^2\, N_{\rm cut}$	$\lvert\hat{f}_b\rvert^2\, N_{\rm tot}$
0.03	0.02202	1.040	2 146	7 782 182
0.05	0.02910	1.009	1 191	6 794 696
0.10	0.04400	0.947	489	6 541 781